



Numeracy Nugget #3 - The Shades of Belief

George Rebane

What does it mean when we hold a belief? We constantly say that we believe this to be the case or that to be the case. Does 'I believe it's going to rain tomorrow' mean that we are certain that it will rain and would bet the farm on it? Or does it mean that we think rain is 'likely' tomorrow? Dictionary definitions of 'believe' are all over the place from "to accept as true or real" through "to expect or suppose" to "have an opinion". We know that to hold a belief does not have to mean that something is absolutely true since we're used to saying things like "Seeing that strengthens/weakens my belief in ...". About all that most people would agree on is that to believe some statement, tenet, or proposition is to believe in a way that indicates to other people that the tenet has a chance of being true.

All of this is in opposition to the notion of 'I know that ... is true' which declares to everyone that you are certain as to the truth of something – "I know that I have three children." – and that you will behave accordingly. By asserting knowledge, the concept of certainty and its related behaviors enter the picture. Scientists in the fields of machine intelligence and complex systems consider the whole matter in terms of degrees of belief with an numerical value ranging from zero to one. Zero denotes that the proposition in question – e.g. it will rain tomorrow – will most certainly not hold. Assigning one to the degree of belief connotes that the proposition will most certainly be true. Values between zero and one express the strength of conviction one has in the proposition being or becoming true. In short, we understand that such 'in between values' are really a measure of uncertainty.

With a few other restrictions placed on this measure of uncertainty, understanding belief in the above terms contributes not only to clear thinking but also supports computation and therefore is usable in (realworld) decision processes that involve multiple propositions. By ascribing various degrees of belief to certain related propositions ('he is a life-long smoker who

has now developed a cough') we can compute the degree of belief we should have in a derived proposition ('he has lung cancer').

One of these additional restrictions is that if the degree of belief in proposition A, noted as $\text{belief}(A)$, is at some value, say 0.7, then the degree of belief in the opposite proposition 'notA' must have the complementary value in the sense that $\text{belief}(A) + \text{belief}(\text{not}A) = \text{belief}(A \text{ or } \text{not}A)$. Since we know from elementary reasoning that a proposition is certainly either true or false, we assign $\text{belief}(A \text{ or } \text{not}A) = 1$, the value of certainty. From the above arguments and the illustrated equation it is now clear that $\text{belief}(\text{not}A) = 0.3$. In short, the complementary propositions A and notA together exhaust the possibilities, This in turn requires that the total contribution of their individual degrees of belief sum to unity, the value of certainty – i.e. one of the complementary propositions must be true.

At this point some readers may say that all this is elementary stuff that everyone knows. Unfortunately that is not the case as witnessed by attending any public meeting and noting the regularity with which people make sincere statements that violate the above principles. This innumeracy then often confounds the intended direction of the deliberations by taking the discourse down back alleys that lead to nowhere. (We all have experienced sponsors of such gatherings who then announce that the meeting still fulfilled the published agenda objectives.)

The less people are exposed to such knowledge about the meaning of belief and uncertainty, the more they have the tendency to see things in terms of absolutes (certain or impossible). They will then gather the wrong data, draw erroneous conclusions, and make bad decisions. Consider the following drivel that all of us have heard uttered by some public official or so-called expert – "Tonight her election is a clear probability.", and "It is no longer just a possibility but has now become a probability." The inanity is here compounded in many listeners who think

that these statements have somehow enlightened them.

Now what does it mean when someone uses the term ‘probability’, and, perhaps, goes on to connect a number like 0.4 or 40% to the term - for example, ‘I think the probability is 0.4 that it will rain tomorrow.’ ? They could be just quoting the TV weatherman and really saying that they believe with certainty that such statements are absolutely reliable. But when probed further about what the weatherman meant by 40%, they ultimately arrive at a conclusion like ‘well, whatever the meteorologists are looking at now, when they saw similar things in the past, four times out of ten the next day it must have rained.’ Or they could come to such a conclusion from checking their own observations and memory. In either event what is happening here is that both assessments fundamentally assume that the underlying weather generating process has not changed. And this is a very important point.

Almost all assessments of future outcomes are based on the constancy of some underlying process. We all assign probabilities or likelihoods using this assumption and then make subsequent decisions without giving ‘process constancy’ much thought. But if someone says something that implies the process may have changed, we instinctively doubt our earlier conclusion and start reassessing the odds. Knowing that our spouse almost always (probability = 95%) arrives home between 6:00 and 6:10PM, we instantly begin reassessing the arrival time upon hearing that there was an accident on the route home. The report of the accident changed our internal model of the process that usually determines the commute time. We conclude that the old process is no longer operational and start estimating a new arrival time based on our knowledge of the relevant delays or possible detours – i.e. we start synthesizing a new underlying process to support our revised belief about when to expect her.

Most people do this kind of thinking and planning ‘naturally’ without understanding the underlying mechanisms. But when we gather to make public policy concerning community growth, health care, and emergency planning, then we would like to do it systematically by taking into account quantified beliefs that are exercised in correct probability models to answer

the necessary ‘what if’ questions. This is called reasonable behavior.

We will take a deeper look at beliefs, probabilities, and randomness in a future Numeracy Nugget. An important idea that we will discover is that there are no objective probabilities (or beliefs), they are all conditioned by the knowledge of the believer. Now let’s go over the homework from NN2 and leave you with something to puzzle over until next time.

Solution to NN2 problem. The ten condemned men were left alone to contemplate their fate at sunrise when one of them came upon the following plan that would perhaps save all of them. He told his compatriots that regardless of the order in which they were to be buried in the sand, all facing in the same direction, the last man who could see all the other hats except his own would say ‘White’ if he counted an even number of white hats in front of him or would say ‘Black’ if that number was odd. (Recall that black and white hats were placed on the heads so that the wearer could never tell the color of his own hat and that each hat color was chosen randomly, say, by the flip of a coin.) Now the unfortunate last man may still be executed instantly if he called out the wrong color, but no other policy would increase his probability for survival above 0.5. However, at this point the other nine men are home free if they understand this policy. If the ninth man hears ‘White’ and sees an odd number of white hats in front of him, he knows that he must be wearing a white hat because that would make the count even for the last man. If the ninth man hears ‘Black’, then the white hat count must be the odd number that he himself sees and, consequently, he must be wearing a black hat (and, of course, vice versa). So the ninth man will always call out the correct color of his hat. The eighth takes into account the ninth man’s correct hat color and applies it to the tenth man’s even/odd assessment, either subtracting a white hat and changing the odd/even designation or leaving it unchanged. Seeing all the remaining seven hats in front of him, he goes on to make the correct call about his hat’s color. To the consternation of the cynical warlord this life-saving algorithm continues to the end of the line. When it is over there will be at least nine very relieved men along with a 50-50 chance of all ten having saved themselves.

The NN3 problem. On a quiz show the contestant is presented with three doors (A, B, and C) behind only one of which there is a valuable prize. The contestant is asked to announce the door which s/he believes to conceal the valuable prize. Before opening that door the host, who knows which door shields the prize, opens one of the unselected doors and reveals to all that the prize was not hidden there. He then gives the

contestant the opportunity to stay with the initially chosen door or to switch and select the other still closed door. Should the contestant switch or stay with the first choice; why? (This famous problem in probability is named after the TV host Monty Hall who posed it to contestants on the popular quiz show 'Let's Make a Deal'.) We reveal the solution next time in NN4.